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EMERGING CFD TECHNOLOGIES AND AEROSPACE VEHICLE DESIGN

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OVERVIEW

With the recent focus on the needs of design and applications CFD, research groups have begun to address the traditional bottlenecks of grid generation and surface modeling. Now, a host of emerging technologies promise to shortcut or dramatically simplify the simulation process. This paper discusses the current status of these emerging technologies. It will argue that some tools are already available which can have positive impact on portions of the design cycle. However, in most cases, these tools need to be integrated into specific engineering systems and process cycles to be used effectively. The rapidly maturing status of unstructured and Cartesian approaches for Inviscid simulations makes suggests the possibility of highly automated Euler-boundary layer simulations with application to loads estimation and even preliminary design. Similarly, technology is available to link block structured mesh generation algorithms with topology libraries to avoid tedious re-meshing of topologically similar configurations. Work in algorithmic based auto-blocking suggests that domain decomposition and point placement operations in multi-block mesh generation may be properly posed as problems in Computational Geometry, and following this approach may lead to robust algorithmic processes for automatic mesh generation.

I. INTRODUCTION

Over the past 20 years, Computational Fluid Dynamics has made significant progress toward generating accurate simulations of flows around realistically complex aerospace configurations. While pundits are quick to point out that there exist multitudes of topologically simple model problems which quickly reveal shortcomings in turbulence models, dissipation models or advection schemes, a widening class of problems has moved within reach. Thus, although some regions of the flight envelope remain outside the realm of affordable and reliable numerical simulation, a growing body of evidence suggests that many critical situations may be predicted with accuracy. As a result of this increased confidence, the past decade has witnessed a shift in the focus of the CFD community from studying flow physics on topologically simple model problems toward ever more bold attempts at simulating vehicles in flight. This shift is evident throughout the military laboratories, NASA and industry as new codes are developed with increasing attention to generality and utility.

I.A Computational Fluid Dynamics in Aerospace Design

From the first studies of numerical techniques for solving the Euler equations by Courant^[1] Lax and Friedrichs^[2], and the landmark calculations by MacCormack^[3], CFD development has centered on issues of accurately solving the governing equations of fluid mechanics. This work set the tone for much of the subsequent development. Implicit schemes for centered spatial operators were presented in the mid 70's by Briley (1975)^[4], and Beam and Warming (1976)^[5]. Jameson *et al.*^[6] introduced a very successful finite volume Runge-Kutta scheme in 1981 at about the same time that Enquist and Osher^[7], Osher^[8] and Roe^{[9],[10]} were beginning development of approximate Riemann solvers which lead to many successful upwind methods in the years that followed.

This brief chronology highlights a major point when one considers CFD applied to the design cycle. While Steger had begun to consider complex configurations as early as 1978^[1], development did not begin to concentrate on design or applications CFD before the mid 1980's. In 1985 Benek, Buning and Steger^[12]

introduced a 3D chimera scheme for application to problems with realistic geometric complexity. This occurred at approximately the same time as other segments of the community pursued multi-block structured solvers and triangular mesh schemes for confronting the same issues^{[13],[14]}.

Euler^[14] and Navier-Stokes^{[15],[16],[17]} computations of flow around complete aircraft began to appear in the latter half of the 1980's. While these represented stunning achievements, they also served as omens which, it may be argued, the community was slow to identify and act upon. Grid systems for some of these early calculations consisted of single block meshes which literally took man-years to develop^{[15],[16],[17]}. Multi-block approaches for Navier-Stokes simulations of complete configurations appeared in 1987^[18]. However, while multi-block was a significant step forward, these efforts were also largely singular and did not directly focus on streamlining the grid generation or surface modeling steps in the process.

The evolving situation was documented in 1992 in an address by Cosner^[19]. Figure 1 is taken from this reference and we use it here to epitomize the experience of designers and applications CFD groups. The figure depicts the breakdown of man-hours required to obtain two Navier-Stokes solutions of the transonic flow around an F/A-18 on meshes with about 3 million nodes. Even with the block structured grid generation tools available to industry in 1992, this chart shows that only about 14% of the man-hours were dedicated to running the solution. Fully 80% of the effort was spent on grid generation and geometry acquisition. Note that man-hours generally represent a direct monetary cost to the work's sponsor.

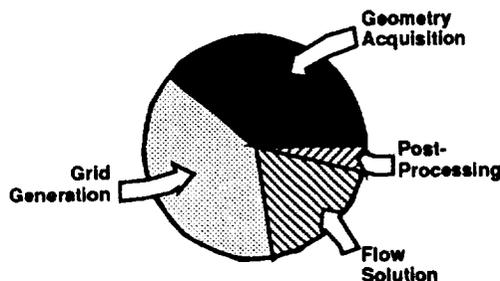


Figure 1. Breakdown of man-hours from CFD solution of F/A-18E/F configuration by McDonnell Douglas in 1992. (Reprinted from Ref.[19] with permission).

In the early 90's, examples like this lead to a better fundamental understanding of the importance of surface modeling and grid generation to the overall of the CFD solution process. However, excessive time for grid generation and surface modeling are not the only roadblocks to CFD's use in design and applied aerodynamics environments. In a well conceived article presented to the AIAA in 1991, Gamer *et. al*^[20]

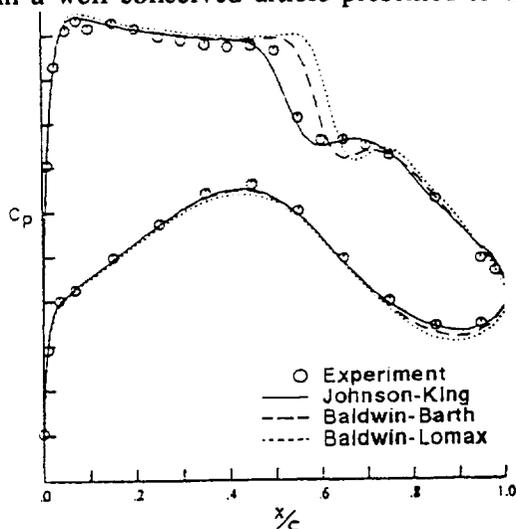


Figure 2. Sensitivity of surface pressure coefficient, C_p , profiles to the choice of turbulence model for a 3D transport wing. (Reprinted from Ref.[20] with permission).

highlighted several fundamental obstacles facing CFD's use within a design process. Figure 2 is an excerpt from this report. These data depict the result of three separate thin-layer Navier-Stokes solutions around a wing, varying only the turbulence model. The figure compares the distribution of the surface pressure coefficient, C_p , resulting from these calculations. In examining these figures, the designer becomes aware of the tremendous sensitivity of the shock location to turbulence model (nearly 10% c variation over the 3 models). Subsequent improvements in the models may reduce this variation somewhat, but the essential problem of discrete solution sensitivity to turbulence model remains.

A similar situation is shown in Figure 3 - excerpted from the same report. This figure highlights the sensitivity of surface pressure to grid resolution at the trailing edge of the wing. These results emphasize that simply generating a mesh (even automatically) is not sufficient. The grid must accurately resolve not only the geometry, but also the physics of the flow. Of course, if a grid system takes weeks to setup, then designers cannot be expected to iterate through multiple cycles of grid generation and flow solution.

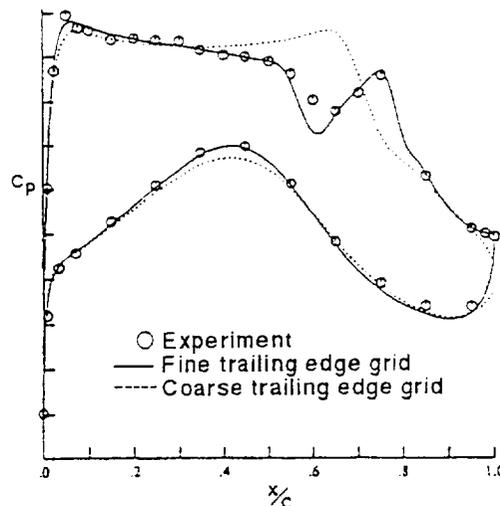


Figure 3. Effects of trailing edge mesh resolution on surface C_p profiles for a 3D transport wing. (Reprinted from Ref.[20] with permission).

I.B. Efficiency of the Numerical Simulation Process

The preceding three figures summarize the main difficulties historically associated with CFD in design; setup time, cycle time, physical modeling, and risks associated with inexpert generation of grids or setting of parameters within codes. These factors all raise the risks (and expense) associated with CFD's incorporation into aerospace design.

The aspect of time deserves special emphasis, both as measured in calendar time and in man-hours. For example, reducing the simulation time could allow multiple cycles to be run and therefore lower the risk involved in the process. Since this cycle is dominated by the human efforts of engineers and scientists, shortening the process generally equates to automation. Such increased automation opens the door for engineers with less specialized training. As a result, motivation exists on many levels to decrease simulation cycle times.

Other aspects of the impact of cycle time on design and applications CFD are less obvious from the perspective of a research environment. In a thoughtful review article presented to the AIAA in 1994, Rubbert^[21] presents an economic model of the aircraft industry which further emphasizes the fundamental importance of time within the design cycle. This model is used to demonstrate that market share is inversely proportional to the time required to produce an aircraft of fixed specifications. Initially it seems intuitive that a company with a slow design cycle will lose market share to quicker, more flexible manufacturers of comparable products. However, the example documented in Figure 1 shows that the promise of reaping the

benefits of full CFD analysis in a design asks that manufacturers adopt a process which is 80% “overhead”. Converting file formats of surface geometry files, computing lines of intersection between surfaces, laying-out grid block topologies, matching faces of grid blocks and stretching ratios of grid points, all represent, essentially, internal steps. They require highly skilled individuals to perform repetitive tasks on expensive equipment, and produce only an intermediate result. After they are used, the intricate details of the process add very little value, unless the experience acquired reduces the overhead of the next project.

Over the past five years, research groups have increasingly focused on increasing the efficiency of the simulation process. This work has revolved around building processes and systems which permit the utility of CFD to be limited by the accuracy of the physical models and numerical algorithms - rather than the time taken to re-grid a configuration when a pylon is moved, or when the aircraft’s outer moldline must be modified due to structural considerations.

I.C. Overview of Emerging Technologies

The techniques reviewed by this paper fall into several categories. *Unstructured* methods include any approach in which the tessellation of the computational domain or sub-domains does not map onto a logical right parallelepiped. This classification is therefore general enough to include methods with grids of pure tetrahedra, hybrid meshes, unstructured hexahedral meshes, and some implementations of Cartesian methods. Also included will be several methods which control or “drive” traditional multi-block grid generators and solvers. Such control based techniques generally proceed through direct automation of the tasks involved in the layout and construction of overset or abutted block structured meshes.

More specifically we will examine:

Pure unstructured – Tetrahedral based approaches for mesh generation and flow solution.

Hybrid methods – Schemes based on mixed elements, including prismatic/tetrahedral meshes intended to resolve viscous near-body layers on prismatic elements, and inviscid regions using tetrahedra (e.g. [22], [23], and [24]).

Cartesian methods – Cartesian, non-body fitted methods, in which the geometry is “cut out” of subdivided Cartesian cells within the mesh. Algorithms for such tessellations may consider the mesh through unstructured, octree structured, or embedded (and i,j,k structured) operations.

Automation / Controllers for Block Structured CFD Schemes

Auto-blocking – Automatic generation of sub-domains and required connectivity based on global information and surface geometry specifications.

Grid Abstractions – Topology of surface geometry is communicated to grid generator through building-block like geometric abstractions.

Scripting – Block layout and grid generation is handled automatically for pre-defined generic topologies. Operating system level controllers also run flow solution and automate post-processing of results (see the article by Buning²⁵ in these proceedings).

Macros – During grid generation, the user’s actions are “recorded” in a macro, which may be played back to generate meshes around other geometries with the same topology.

A survey paper such as this is based upon a subjective view of the research it discusses. Inevitably, the author’s opinions and experiences filter the material and flavor the discussion. Nevertheless, this paper attempts to present a valid perspective on the issues discussed, and concentrate on pertinent topics of general interest and concern.

II. EMERGING TECHNOLOGIES

This section presents overviews and assessments of promising techniques which are aimed at reducing the time required for numerical simulation. Rather than attempt an exhaustive review of each topic, the discussion will lead through a general overview supported with important results and discussion stemming from analysis within the research community. Since these technologies are in various stages of maturity, an

attempt has been made to document the current status and summarize the potential outcome of future research. Subsequent sections will use this information to investigate ways in which these methods can be effectively used within the design and applications CFD environment.

II.A. Pure Unstructured Methods

Techniques involving unstructured grids composed of triangles (in 2D) or tetrahedra (in 3D) have been applied to the Euler and Navier-Stokes equations since the mid- 1980's.^[26] Based upon theoretical foundations developed largely in the late 1970's^{[27],[6]}, such methods have been attractive from the outset since their spatial discretization operators avoid unnecessary assumptions about global mesh topology. Thus, solvers view the mesh as an unstructured collection of cells, with some explicitly defined connectivity. The grid generation task reduces to the more general problem of generating and tessellating coordinate data between prescribed boundaries, and is therefore more amenable to automation. This section will detail the current state of such research and discuss approaches for flow simulation and mesh generation for inviscid and viscous simulations.

II.A.1 Unstructured Flow Solvers

Unstructured simulations using central difference finite volume techniques were applied to the inviscid simulation of a complete aircraft as early as 1986^[26]. The short span of time from inception to application on such complex configurations provided significant motivation for their continued development. Several outstanding reviews and summaries exist to track development of these methods over the past decade. Excellent articles have been prepared by Venkatakrishnan^[28] and Mavriplis^[29] chronicling progress in both flow solver development and mesh generation. In addition, the proceedings in reference [30] provide a detailed description of a variety of unstructured methods with supporting theory and applications, including domain decomposition for parallel architectures, implicit time integration, and a host of mesh generation strategies.

The first flow solvers of Jameson and Mavriplis [26] operated in a cell-centered finite volume environment. In the subsequent 3D work, which appeared in 1986, Jameson *et al.* [14] extended these ideas to permit vertex based formulations and related the central difference finite volume framework to a Galerkin finite element approach with linear basis functions. Desideri and Dervieux^[31] presented one of the first higher-order accurate upwind schemes for triangular meshes in 1988. This work relied upon van Leer's MUSCL^[32] scheme to construct a node-based upwind method on meshes composed of simplices. Since unstructured meshes have no dominant coordinate direction, some ambiguity existed in the implementation of one-dimensional limiting ideas and the use of directional operator splitting on unstructured meshes^[33]. This shortcoming was overcome when Barth and Jespersen^[34] presented an unstructured upwind method based on a central difference gradient estimation that obeyed multi-directional monotonicity principles.

In the past five years, a host of improvements have been proposed to many of these algorithms. Just as importantly, however, theoretical frameworks have been introduced which give insight into commonality between various schemes and aide in developing efficient implementations. For example, reference [35] proposed the use of edge-based formulas and data structures which have since become one of the major contributions of the research. Since that time, a variety of authors have invoked minimal storage and computational complexity arguments to further support the use of such structures in node based schemes [35],[36],[37].

Most unstructured Navier-Stokes work has adopted either central difference finite volume or finite element discretizations of the diffusive terms in the governing equations (see for ex. [35], [38], [39], [40], among others). Like the convective terms, these operators are also amenable to edge-based data structures which permit compact construction and storage. Reference [28] provides additional sources of discussion concerning recent developments in unstructured Navier-Stokes solvers.

II.A.2 Mesh Generation

Initial efforts with unstructured techniques generally relied upon meshes constructed from quadrilateral structured meshes through the addition of a diagonal spanning each cell^[26]. In their initial complete aircraft calculations, however, Jameson *et al.*^[14] began to invoke principles of Delaunay triangulation in tessellating point data and related techniques have become commonplace in subsequent mesh generation strategies. This initial work identifies the main strength of unstructured approaches - that is, the ability to automate generation of a volume mesh from an initial triangulation of the domain boundaries. Thus, the tedious, ambiguous, and often complex process of decomposing the computational domain into regular hexahedral blocks is avoided.

A Delaunay triangulation of coordinate data is one in which the circumcircle through the vertices of each triangle does not contain any other vertex in the mesh^[41]. Most approaches can be classified as either Delaunay, advancing front^{[42],[43]}, or a combination of the two^{[44],[45],[46]}. Recent research in this field has produced a variety of *Steiner Triangulation* techniques^[47], in which new sites are incrementally inserted into existing, usually Delaunay, triangulations^{[48],[49],[50]}. Such methods appeal to the unique properties of Delaunay triangulations^[51] to produce meshes with mathematically provable control over maximum angles and mesh quality. For example, a Delaunay triangulation of a planar point data satisfies the min-max criterion, *i.e.* it leads to a unique graph which minimizes the maximum angle of any triangle in the mesh. Exploitation of this property, taken in conjunction with a circumcenter site insertion strategy permits advocates of Steiner triangulation techniques to produce triangulations which guarantee that all angles in the final mesh will lie between prescribed minima and maxima^[52].

The phenomenal success of references [42], [44], [53], and others, attests to the fact that two and three dimensional meshes of isotropic triangular or tetrahedral elements can be robustly generated with well documented methods. Current research is therefore directed at generation meshes containing stretched elements in a similarly reliable manner^{[46],[49],[50],[54]}. Although many techniques have been proposed, this topic remains one of the pacing items in the application of unstructured Navier-Stokes solvers to complex configurations.

II.A.3 Current Status

The recent literature has witnessed a virtual explosion of research in unstructured methods. However, looking with an emphasis on design and applications CFD, we are able to consider a smaller subset of this work which is restricted to 3D applications. Within this subset, several classifications still exist. Research pursuing efficient and accurate Navier-Stokes solution represents a large body of work and several competing methods are being debated. Nevertheless, applications are not yet commonplace, and one may argue that this work is still formative. Similarly, despite some impressive results^{[55],[56],[57]}, time dependent adaptive schemes and re-meshing algorithms such as those for unsteady flows and moving body problems, is also largely a research topic. Excluding these topics we are left with work which presents applications of unstructured methods to 3D steady flow fields.

This process of narrowing down and classifying ongoing research is instructive. It emphasizes the point that although the research community exerts considerable resources in unstructured flow solvers and mesh generation, only a small fraction of this effort is dedicated to working problems directly related to the generic application of these methods. Relatively few researchers focus directly on issues surrounding efficient implementation and routine application. Such work is, perhaps, almost unattractive in an environment which rewards novelty and fundamental concepts. However, from an industrial or applications CFD point of view, the automation of routine processes is of paramount importance, and in this polarity lies a serious conundrum.

Within the subset of steady, inviscid applications, huge progress has been made since the first complete aircraft simulations of reference [14]. Good examples of relatively efficient codes may be found

throughout the recent literature^{[40],[43],[37],[58]}. Figure 4 contains representative results from this body of work, and was taken from the results of Mavriplis and Venkatakrishnan [58]. The figure depicts a Low Wing Transport (LWT) configuration with pylons and nacelles computed on a mesh of 804,056 vertices connected by a graph that forms 4.5M tetrahedra. The flow is transonic at Mach 0.77 and an angle of attack of 1.116° . This is a good example for the present discussion, since the code was written to take advantage of edge-based data structures, and requires only approximately 150 words of storage per vertex (relatively few for an unstructured multigrid code). Convergence acceleration of the multi-stage Runge-Kutta time stepping scheme is performed through agglomeration multigrid^[58] which generates coarser meshes by fusing together fine grid cells. This multigrid strategy is particularly well suited to complex geometry, since it avoids the traditional shortfall of failing to resolve the geometry on coarser meshes in the grid sequence.

Figure 4a shows a view of the surface grid for the first coarser mesh in the multigrid cycle (containing 106,064 nodes). Figure 4b contains surface and symmetry plane Mach contours showing the lambda shock structure on the wing, and the effects of propulsion integration. The convergence history depicted in Figure 4c indicates that the L_2 norm of the residual was reduced six orders of magnitude in 100 multigrid cycles.

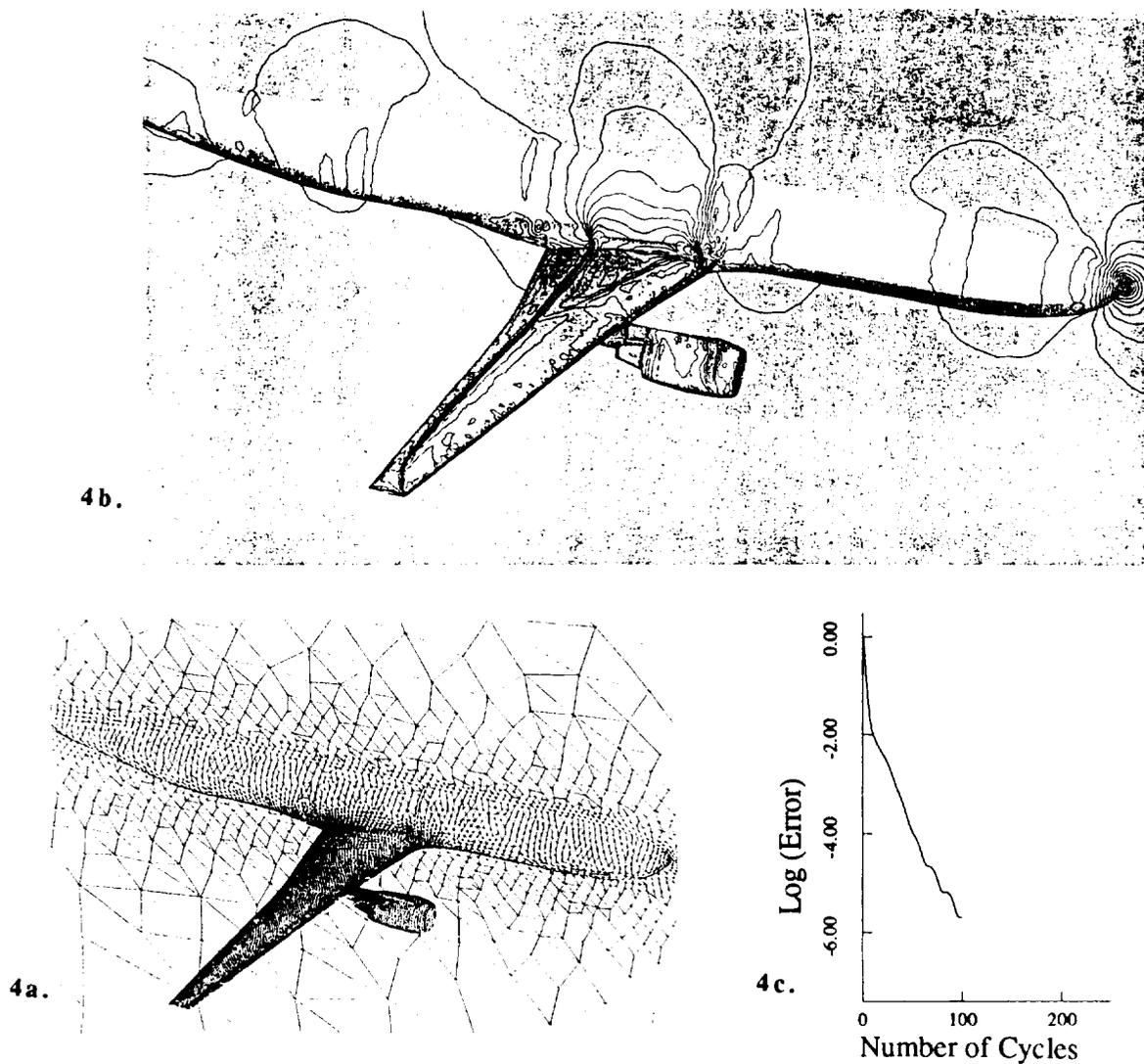


Figure 4. (a) Surface mesh for coarse grid with 106,064 vertices, fine mesh (not shown) contains 804,056 vertices. (b) Surface and symmetry plane Mach contours for transonic flow over LWT configuration. (c) Convergence history of 7-level agglomerated multigrid for LWT case. (Frames reprinted from reference [58] with permission).

The simulation required 96MW of memory and 90 minutes of single processor CPU on a Cray Y-MP. These results are representative of the CPU and memory requirements of modern, well-written unstructured solvers.

II.A.4 Unresolved Issues and On-going Research

Surface modeling – In the context of unstructured methods, surface modeling plays two important roles. First, an initial triangulation (which is constrained to the surface geometry) must be supplied to the volume mesher for generation of the initial mesh. Secondly, if flow field driven adaptation is permitted, new sites may be inserted on the boundary and these too must conform to the actual surface geometry. Generation of such a constrained initial surface triangulation is non-trivial when one considers that the surface may be originally described through CAD data, loftings, natural NURBS, trimmed NURBS, patches or surface grids. Edges in this triangulation must follow leading edges, trailing edges, junctures between components, and may not be allowed to swap (if edge swapping will produce faces which no longer coincide with the surface description). Generation of this surface triangulation is typically the most time consuming and user-intensive operation in the mesh generation procedure. While volume meshing may take minutes, or tens of minutes on an engineering workstation, the initial triangulation may take hours to days. Moreover, if the surface triangulation is computed with a preprocessor or commercial software package, the true geometry description may be unavailable for later use, and adaptation becomes a clumsy process.

Anisotropic Mesh Generation – Several approaches have been proposed for creating meshes with high aspect elements suitable for Navier-Stokes including those in references [54], [59], [46], and [50] among others. However, research involving the retention of angular control while still retaining a sufficiently smooth blending of anisotropic viscous cells with isotropic Euler elements continues. In addition, the insertion of high aspect ratio tetrahedra into general unstructured meshes increases the likelihood of generating “sliver” elements, non-tetrahedralizable regions, etc. Such effects have a detrimental effect on the robustness of viscous grid generators. Finally, even with the addition of solution based adaptation, difficulties exist concerning meshing wakes and other free shear layers with sufficient resolution^[54].

Counting Arguments – A variety of authors have raised concerns about the total numbers of cells and edges required to fill volume space with high aspect ratio tetrahedra. As an example, suppose we seek to grid the boundary layer region around a single element 3D wing at elevated Reynolds number. A structured surface mesh may contain 200 vertices around the chord and perhaps 50 spanwise stations. Spanning an expected attached boundary layer with 25 stations would permit sites at y^+ of around unity with reasonable grid stretching. Thus the boundary layer portion of the mesh would consist of $200 \times 50 \times 25 = N = 250,000$ vertices, and hexahedral cells made up of 750,000 edges (assuming we remain away from mesh boundaries). Tessellating this structured region with right tetrahedra would lead to dissecting each hexahedral cell into $\alpha = 5$ tetrahedra (best case). This leads to 1.25M tetrahedrals and $(\alpha+1)N = 1.5M$ edges. Thus, for an explicit, edge-based, code we have doubled the storage and *per sweep* CPU requirements simply by virtue of the tessellation. In an implicit code the edges must still be stored, but now the additional connectivity implies a fuller matrix for inversion. Moreover, recent analysis and numerical experiments suggest that for certain classes of schemes, the additional diagonal edges do not necessarily lead to greater accuracy^[60]. (In fact, Ref. [61] has actually demonstrated improved discrete solutions through choosing control volumes which de-emphasize the contributions of such diagonal edges.)

II.B Hybrid Methods

The counting arguments, accuracy concerns and other issues in the preceding paragraphs have prompted several groups to consider alternatives to pure unstructured methods [22],[23],[24],[62],[63]. For example, reference [60] suggested that diagonal edges found in semi-structured viscous layers of triangular or tetrahedral meshes could be deleted to leave triangular, hexahedral or prismatic elements. Such edges incur a memory penalty with no apparent accuracy advantage and thus degrade the efficiency of the method. The

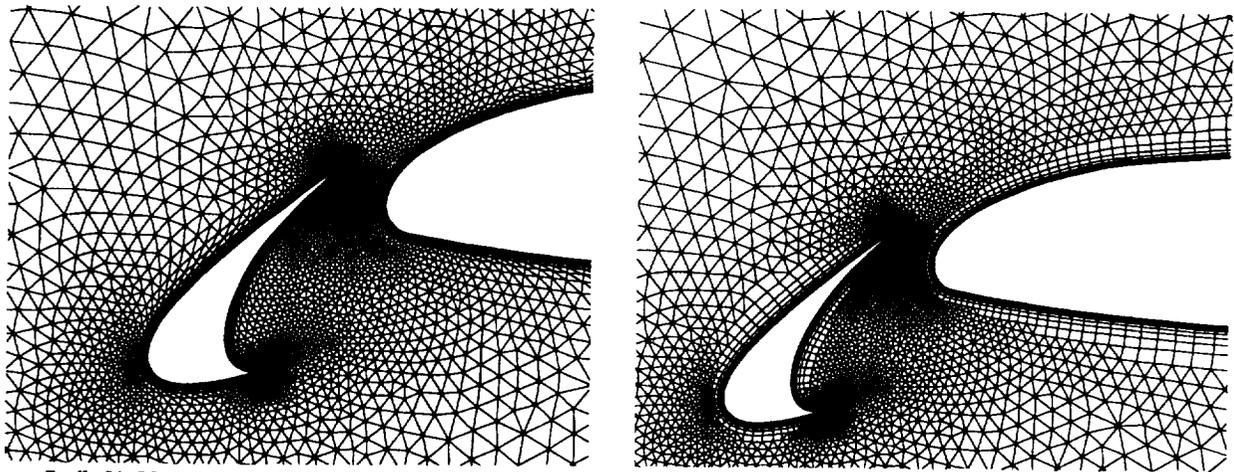


Figure 5. (left) Unstructured mesh near the first and second elements of a multi-element airfoil discretization. (right) Mixed element mesh where 11,544 triangles were collapsed to form 5,772 quadrilateral elements. (Frames reprinted from reference [46] with permission).

extension of edge-based unstructured flow solvers to accept hybrid elements is relatively straight forward and the idea has been investigated by several authors. Figure 5 contains two figures excerpted from the work of Marcum[46] and shows just such an approach. Sites forming triangles in the semi-structured wall boundary region near the leading edge of a multi-element airfoil have been re-tessellated as quadrilateral elements. In this example, 11,544 triangles near the wall were collapsed to form 5,772 quadrilaterals through removal of the unnecessary diagonals. In addition, reference [46] has investigated collapsing tetrahedral elements to form prisms in semi-structured regions of the mesh, but final results have not yet been presented.

II.B.1 Prismatic and Hybrid Meshes

The use of prismatic elements for efficiently discretizing viscous layers, or even entire domains has received increased attention since the late 1980's. Prismatic mesh generation ideas have been presented by references [23], [24] and [62] among others. The methods proposed by Nakahashi[23] and Kallinderis[62] are based on algebraic construction principles that begin with a surface triangulation of the geometry. Families of mesh lines spawn from mesh vertices in the surface triangulation. The mesh is constructed layer by layer in a manner which is analogous to an inflation of the surface triangulation. This construction technique permits a semi-structured interpretation of the mesh. The lines emanating from vertices on the body are continuous, and nodes along each of these are vertices of fixed degree. This local structure permits straight forward implementations of semi-implicit multigrid, or even semi-coarsened multigrid convergence acceleration[24],[64].

The algebraic marching procedure described here clearly has the character of a hyperbolic algorithm for the positioning of sites on subsequent layers of the mesh. Recognizing this, Pandya[24] has proposed alternative construction schemes based on solving hyperbolic partial differential equations for determining the vertex locations on successive mesh layers.

One criticism of early prismatic mesh generation techniques was that the extension to multiple body configurations was unclear, since there appears to be no clearly defined manner in which to prevent multiple prismatic meshes to fuse together as they grow. To address this issue, recent work presented by Kallinderis[22] proposed constructing layers of prismatic elements only near the wall boundaries to resolve viscous layers, and then discretizing the rest of the domain using an advancing front technique, filling the space with isotropic tetrahedra. If prismatic layers from multiple elements overlap, then they are locally "receded", and the space in between may be filled with tetrahedral elements during the advancing front step. Reference [22] presents a variety of viscous multi-body meshes constructed following this procedure.

II.B.2 Current Status and Unresolved Issues

As compared with unstructured tetrahedral techniques, research into prismatic and other hybrid methods is relatively new. As a result, a smaller body of work exists to highlight their strengths or document potential shortfalls. Research in these methods has shown great promise, however there are still relatively few detailed computations on sufficiently fine grids.

Hybrid methods were developed in direct response to the concerns about anisotropic tetrahedra and counting arguments documented above, and they have successfully overcome many these obstacles. The research community has been quick to respond to criticisms concerning multi-body and meshing concave domains and is similarly working to resolve most outstanding issues. A partial list of potential roadblocks includes:

Surface modeling – Like pure unstructured methods, prismatic techniques and most other hybrid methods generally employ a surface triangulation to begin the mesh generation process. Thus, such methods inherit the difficulties associated with generation of such a triangulation that were presented, above, in section II.A.4

Adaptation – In hybrid techniques that rely upon the semi-structured nature of prismatic regions, the incorporation of adaptive mesh algorithms can become unclear. Parthasarathy *et al.*^[64] have presented some promising work addressing this topic in their consideration of viscous flow around a sphere. In this work, the sphere was encased in a prismatic mesh to capture viscous phenomena and a tetrahedral mesh was employed away from the body. The hybrid adaptation scheme included *h*-refinement and mesh re-distribution. Nevertheless, the flexibility of mesh re-distribution algorithms on more general topologies remains an open question. Additionally, if a tetrahedral cell at the prism-tet interface is *h*-refined the entire stack of prismatic elements between this cell and the body must also be *h*-refined if one is to preserve the semi-structured nature of the prismatic layers. Such a strategy raises efficiency concerns. Of course, if the solver does not make use of the structure in the prismatic layers, and considers the mesh simply as an unstructured collection of mixed element types, the restrictions on adaptation largely disappear. Finally, we note that since prismatic meshes have an *O-O* topology, mesh adaptation to resolve wakes and/or free shear layers within the flow may present a problem

II.C Cartesian Mesh Methods

Cartesian approaches differ from those presented in the previous sections by virtue of the fact that they make use of a *non*-body fitted mesh system to discretize the computational domain. The hexahedral cells in the domain are a set of right parallelepipeds and the original grid system may extend through solid wall boundaries in the computational domain. The process then removes any fully internal cells, flags cells which intersect the body and treats remaining cells as general volume mesh control volumes. The promise of the technique stems from the fact that it trades the case specific problem of generating a body fitted mesh (unstructured, structured, blocked, etc.) with a more general problem of computing and characterizing geometric intersections between Cartesian flow field cells and the surface geometry. Thus, all difficulties associated with meshing a particular geometry are restricted to a lower order manifold which constitutes the outer shell of the geometry - rather than occurring throughout the computational domain. Recent research has demonstrated that these mesh generation operations readily lend themselves toward automation and the method can be applied to extremely complex geometries^{[65],[66],[67],[68],[69]}. Since solid wall boundaries may cut arbitrarily through the layer of “cut cells” encasing the body, surface boundary conditions are of obvious importance in Cartesian schemes.

Cartesian approaches fall into two general categories. Either they consider the mesh to be an unstructured (or octree structured) collection of *h*-refined hexahedra, or they operate by embedding structured sub-grids within existing structured blocks. In unstructured or octree methods, volume meshing of the computational domain relies on a simple and robust procedure of cell division. Beginning with a coarse background grid,

or even a single root cell, the hexahedral elements are recursively subdivided in order to resolve the geometry and evolving flow features. The final mesh is a collection of cells at various levels of refinement, viewed as either entirely unstructured or as having an underlying octree connectivity. Structured approaches embed i,j,k structured sub-grids similarly aimed at resolving geometric and flow field features. Successful 2D and 3D solution procedures have been proposed following both implementations^{[65],[67],[70],[71]}.

II.C.1 Development of Cartesian Approaches

Although three dimensional applications to complex geometry have only recently become commonplace, Cartesian approaches have been evaluated since the late 1970's. Work by Purvis and Burkhalter^[72] solved the full potential equation on 2D Cartesian meshes using a finite volume method. Solution of the Euler equations was pursued in the mid-1980's by a variety of researchers^{[73],[74]}, and the first three dimensional inviscid solutions appeared in the late 1980's by Gaffney, Hassan and Salas^[75].

Cartesian approaches have been successfully utilized in industrial applications including Boeing's TRANAIR code which solves the full potential equation, and the commercially available MGAERO^[76] package for Euler simulations. These applications are notable because they provide close links with surface modeling and provide a wide base of experience with large-scale computations using Cartesian methods. MGAERO, for example, adopts a component based approach to complex geometries^{[77],[78]}. Similar approaches have been adopted within the research documented in References [66],[67] and [79].

The cut cells necessarily present in Cartesian discretizations present unique problems in the implementation of accurate boundary conditions. References [80],[81],[82] and others present insight into the important issues involved.

The isotropic elements stemming from h -refinement of Cartesian hexahedra are well suited to resolving flow structures in inviscid simulations. However, use of such elements to capture boundary layers and other stiff viscous phenomena would be grossly inefficient^{[83],[84]}. Recently, a variety of authors have proposed alternate techniques for extending inviscid Cartesian approaches to viscous flow^{[68],[84]}. Historically, the lack of a clear extension to viscous simulations has been a weak link and the recently proposed venues offer the possibility of further research.

II.C.2 Current Status

In the recent literature, one finds a multitude of work stemming from Cartesian mesh methods. Many notable 3D computations have investigated the flow around complex geometries. The 3D Reynolds averaged Navier-Stokes results which use prismatic meshes near the body in reference [68] are particularly promising. However, such examples are still formative, and the technology has not spread across the community for independent confirmation. The target of most Cartesian work is a role within the inviscid design/analysis cycle. Appropriately, this section restricts its attention to the discussion of 3D Euler applications similar to those found in [67],[69],[78].

While unstructured and structured CFD approaches both involve surface modeling issues, the static nature of their surface description permits these techniques to develop sufficient surface meshes in preparatory steps. A notable exception, of course, are unstructured adaptive algorithms which must interrogate the surface database during each mesh adaptation phase. For similar reasons, Cartesian approaches demand that the mesh generation and adaptation algorithms be closely coupled with the geometric database. Throughout the mesh generation process, Cartesian cells must be constantly tested against the surface database. Again during adaptation, new cut cells are clipped against the surface, and fully internal cells are eliminated. In reference [66], tests were performed with direct inquiries to a NURBS description of the body. However, speed, robustness and other issues motivated a return to the use of a surface triangulation database in later research. Important differences exist between the surface triangulations required for Cartesian and fully unstructured or prismatic approaches. In Cartesian approaches, individual components of the geometry may be independently triangulated into closed polyhedra, without regard to overlap or intersection of the

component triangulations. Thus, the technique avoids computation of surface-surface intersections, and since Cartesian cells may cut arbitrarily through the geometry, they may intersect the triangulations of several different components without special handling. In essence, the actual *topology* of the full configuration need never be communicated to the grid generation routines, and herein lies the fundamental difference with structured or unstructured techniques. This single factor is perhaps the key in understanding Cartesian claims for rapid turn-around and automation. Within many mesh generation codes, it is relatively easy to obtain surface grids or triangulations for individual components if one does not worry about intersections with other parts of the overall geometry. As a result, the geometric modeling is substantially quicker for many applications.

Figure 6 contains an adapted Cartesian mesh generated by Aftosmis and reprinted from Reference [67]. The domain includes over 2.9M Cartesian cells and the surface geometry is described by 12 separate component triangulations. The final adapted grid contains 10 levels of cells and approximately half of these are at the finest level of refinement. The discrete solution began on an initial, geometry adapted, mesh with 5 levels of cells. Further cell refinement was based on the evolving flow solution. Figure 7 shows a close-up of the adapted mesh painted with isobars of the discrete solution on a plane located between the fuselage and first nacelle.

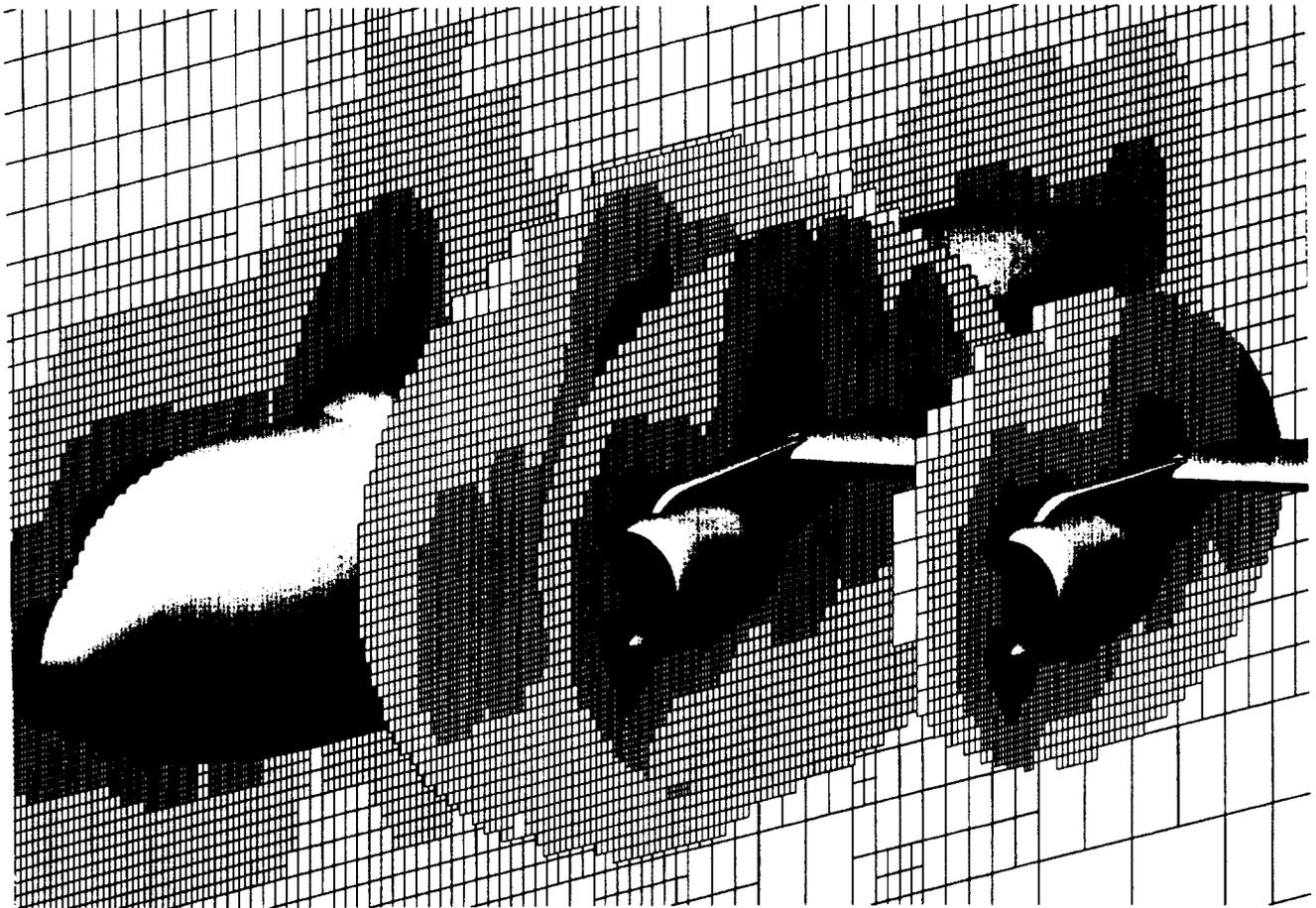


Figure 6. Adapted Cartesian mesh for High Wing Transport (HWT) configuration with 2.9M Cartesian cells and 10 levels of refinement. The surface geometry is described by 12 separate component triangulations. (Reprinted from Reference [67])

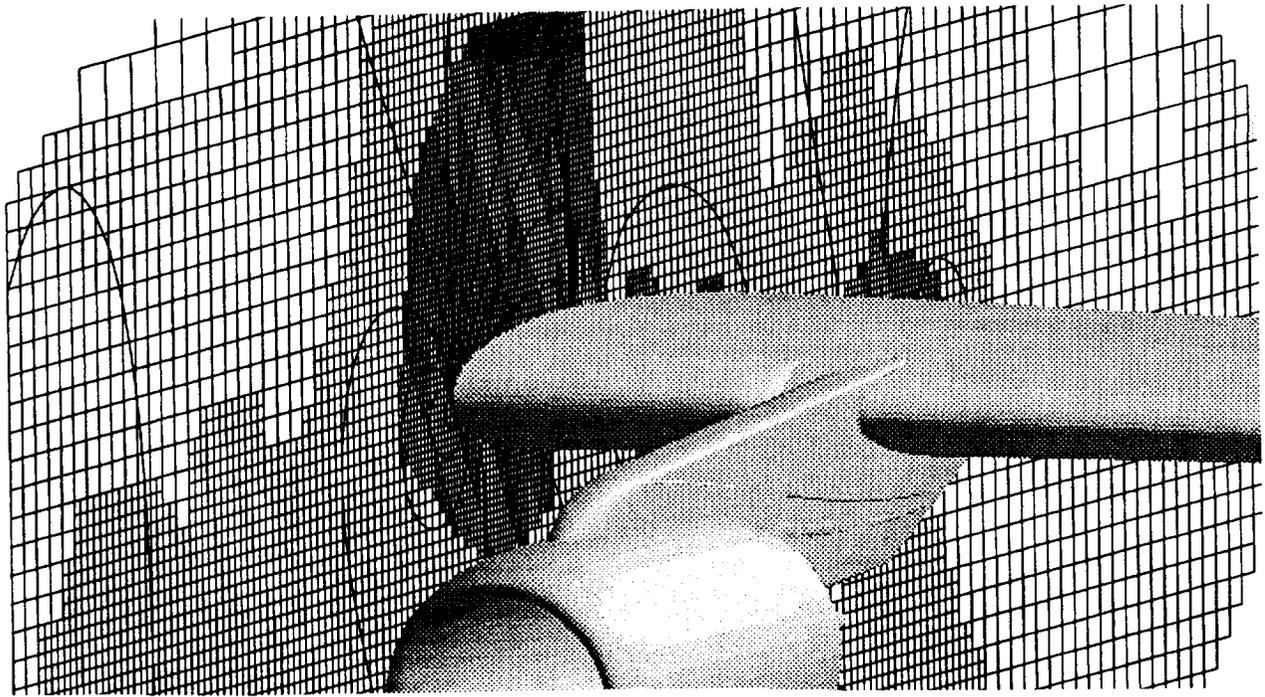


Figure 7. Close-up of the adapted mesh and isobars in the discrete solution on a plane located between the fuselage and first nacelle. (Reprinted from Reference [67])

In an effort to more clearly distinguish automated from interactive tasks in obtaining such solutions, it is useful to trace the primary steps in the process. The example used here is similar to the HWT configuration shown in figures 6 and 7, but with the addition of a winglet, and high lift devices (inc. leading edge slat, flap, flap vane, and spoiler). Figure 8 summarizes the contributions of the various steps as a fraction of the total time required for performing the complete simulation and analyzing the results.

CAD (18%) – Perform CAD operations to generate trimmed NURBS surface description of each component in configuration. *Interactive*

Surface Grid (18%) – Fix problems with NURBS representation, generate surface triangulation and close the triangulation of each component to form watertight polyhedra. *Interactive*

Initial Mesh (1%) – Determine extent of computational domain, and generate geometry adapted Cartesian starting mesh. *50% Interactive, 50% Automated*

Adaptive Euler Solution (40%) – Use of UNIX level control scripts cycle between running the Euler flow solver and flow field adaptation steps. Scripts handle job control, job submission and posting of intermediate results for review by engineer. Note that approximately 80% of this

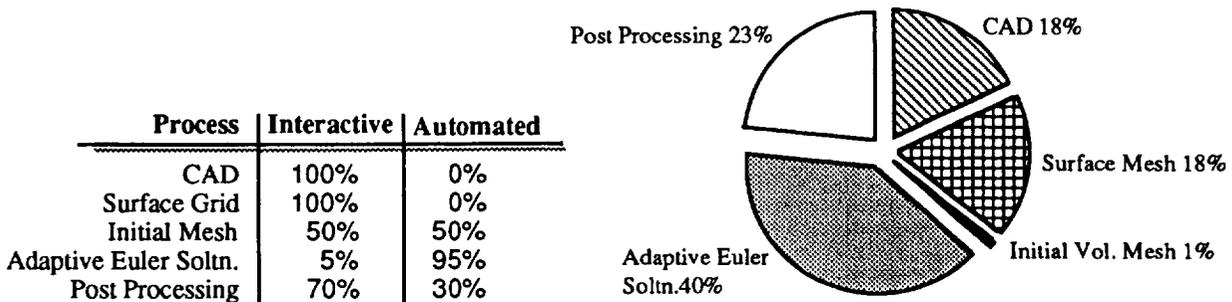


Figure 8. Breakdown of interactive and automated processes in HWT example in figure 9.

time is dedicated to waiting for jobs in queue and not CPU. Also, this code is essentially unaccelerated and therefore converges slowly. *5% Interactive, 95% Automated.*

Post Processing (23%)– Post processing of results to extract surface pressure, cutting planes, streamlines in solution and other structures in flow. Surface pressures and cutting planes are extracted automatically. *70% Interactive, 30% Automated.*

An overall view of the discrete solution for this case is depicted by Figure 9. The computation placed 1.65M cells in the domain, and the adaptation was restricted at the finest level of adaptation to focus on the high-lift system. The figure shows a macroscopic view with isobars, streamribbons and an inset frame detailing the Cartesian mesh surrounding the flap system.

The total time for this simulation, in a research environment without dedicated CAD support or computing resources, was 17 days. This is relatively quick for a research application, but still far from optimal for preliminary design or loads estimation environments. One strength is that subsequent parametric studies, (e.g. flap deflection or nacelle modifications) may be computed comparatively quickly. After approximately 1/2-1 day setup, follow on numerical simulations would turn around in a few days time.

II.C.3 Unresolved Issues and Current Research Topics

Viscous Simulations - Despite increasing documentation of efforts proposing full Navier-Stokes

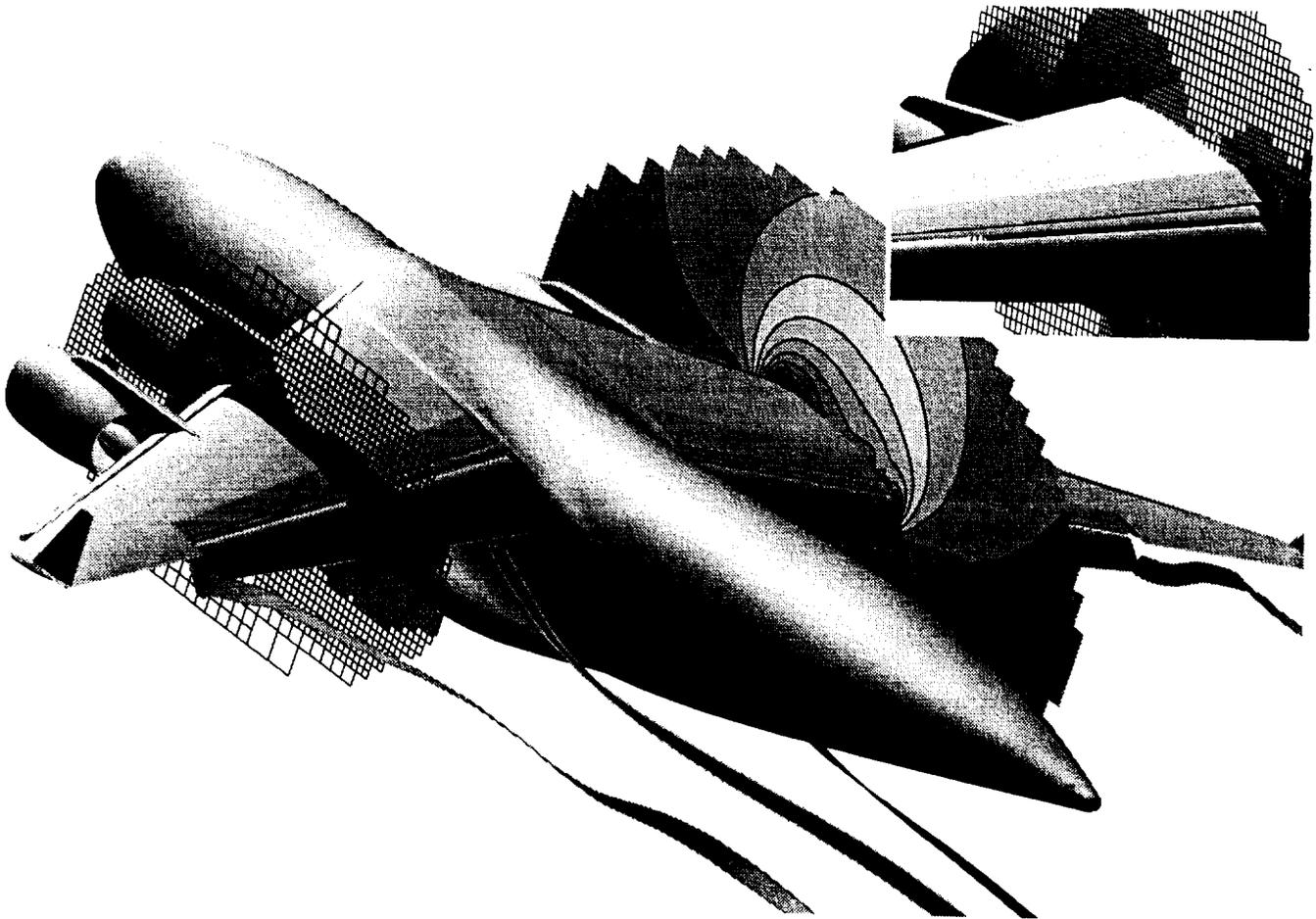


Figure 9. Isobars, mesh planes and streamribbons from High Wing Transport (HWT) case computed with flap, flap vane, spoiler, leading edge slat, winglet, pylons and nacelles. The final mesh contained 1.65M cells at 10 levels of refinement. The inset frame shows a detail of the computational mesh around the flap system.

simulations^{[68],[83],[84]}, open questions exist concerning efficiently constructed methods for viscous solutions on Cartesian meshes. Resolving boundary-layers with isotropic Cartesian cells is inherently inefficient. The use of prismatic grids near bodies inherits the difficulties associated with prismatic methods while adding the complexity of coupling the Cartesian and prismatic cells. As new techniques for constructing surface triangulations for entire configurations become more accessible, this path may prove fruitful. However, adapting Cartesian meshes to wakes and other viscous phenomena will continue to be a problem. Nearer term results may come through the coupling of Cartesian solvers with 3D boundary layer models.

Surface Modeling - Although surface modeling problems with Cartesian methods are somewhat alleviated by working directly from NURBS or triangulations of individual components, the example presented above shows that this procedure still demands considerable effort. Dedicated CAD systems depend on experienced users. Appropriate surface triangulations of various components are similarly non-trivial and require specialized software. Although no serious technical roadblocks exist in this process, the topic requires additional research attention and increased automation.

Accuracy at Boundaries - One view of Cartesian methods is that they take advantage of simpler flow field discretization stencils but suffer from increased boundary complexity. The intersection of the body with the grid may be very arbitrary thereby making the implementation of accurate and conservative boundary conditions more difficult. Although a variety of implementations have been reported, definitive formulations have been elusive. Most work has resulted in implementations that are between first and second-order accurate^{[71],[79],[81],[82]}.

II.D. Automation of Block Structured Mesh Generation

Even with recent advancements in unstructured, hybrid, and Cartesian technology, the vast majority of CFD applications are computed with structured CFD schemes. This is especially true for Navier-Stokes simulations where the robustness, speed, and accuracy of unstructured viscous approaches have yet to be convincingly demonstrated. The basic paradigm of multi-block approaches begins with a decomposition of the computational domain into a collection of deformed hexahedral blocks with either matching or overlapped faces. During the flow solution, the solver processes these blocks independently and then passes information to neighboring blocks as boundary conditions. Modifications of this basic model exist for both parallel machines and time dependent simulations.

In a recent paper, Dannenhoffer^[85] separates the development of 3-D block structured grid generation into three stages of evolution. Initially, grid generation research focused on operations within each mesh block. Conformally mapped, elliptic and hyperbolic methods were developed for creating mesh block, were widely used^{[86],[87],[88]}. Block-to-block pointers, common edges, and all other connectivity or control information was input by hand. Such systems typically operated in batch mode. With the fundamental algorithms identified, research focus then shifted toward usability. Second generation mesh generators replaced the tedious and error-prone process of typed-in control information with input through a Graphical User Interface (GUI) and such systems make up the bulk of currently available grid generators. Such interfaces are designed to give feedback to the user while aiding in visualizing the grid creation process. The end result, however, is essentially the same user-specified location and connectivity information required by the first generation systems. Third generation systems involve automating the blocking and mesh generation process to reduce the dependence on human input and decision making.

In the past decade, a number of approaches have been presented for automating the mesh generation process. These attempts may be generally categorized as either *interactive*, *knowledge-based*, or *algorithmic*^[89], and this section presents a sampling of the current state of research in each of these directions.

II.D.1 Automation of Interactive Processes

The most direct approach toward automation of structured grid generation is through automation of the processes which exist in current gridding strategies. This implies that the user must still ultimately convey the topology of the body and block connectivity, and improvements stem from reducing the clutter of trivial tasks. Bookkeeping and obvious choices may be made automatically by the system, leaving the user free to work on the core problems of mesh and surface topology. One basic algorithm for interactive construction of blocked grids might be as follows:

- Algorithm I:**
- a. Generate and link curves to form block edges.
 - b. Construct block faces from edges.
 - c. Distribute points on block edges and enumerate block connectivity.
 - d. Generate volume mesh within each block.

Some current systems take small steps toward automating this process by automatically dimensioning and distributing points on matching block faces, or by using geometric proximity to infer a topology and/or connectivity. However, even with such subtle streamlining, the process remains highly interactive and cumbersome. The essence of the problem is detail. As long as it requires an engineer to specify each curve of every block, the process will be labor intensive for complex configurations with many hundreds of blocks.

One solution to this problem of excessive detail comes through the classical process of abstraction. Using a technique pioneered by Allwright^[90] the user focuses on generating a blocked *abstraction* of the geometry which then communicates the topology of the configuration to the mesh generation system. Reference [85] has extended these ideas and applied them to a variety of published examples. One approach uses hypercube building blocks to construct “squared-up” abstractions of the configuration in three-dimensions. The use of the hypercube - which is simply the volume between two nested cubes - was first proposed by Allwright and applied to Euler simulations of a complete business jet as early as 1988^[90]. Figure 10 shows a sample abstraction for the HWT configuration of Figures 6 and 7.

Once such an abstraction is formed, the blocking process occurs automatically following pre-set rules and point distribution algorithms. Next the surfaces of the abstraction may be deformed to match their counterparts in the real geometry. These deformations are distributed throughout the blocking, and thus the final block locations and topology are established. The last stage of mesh generation creates the volume grid within each block, using standard techniques.

The fundamental simplification in this approach is that the mesh topology is generated from an abstraction and not the actual geometry. This construction communicates only the *essential* geometric and topological information required to layout the mesh blocks. In addition, the process of forcing the user to construct the configuration out of pre-defined building blocks links certain surfaces with specific block layouts. In this way, the system guides the user, interactively, through the steps necessary to layout the mesh blocks. It is not clear if the procedure described will always lead to the most efficient blocking for any given configuration. However, the strength of the technique is that it will reliably lead to a decomposition. Ultimately this robustness and agility may provide shorter complete cycle times and yield a more efficient CFD process.

Variants of this approach were documented as early as 1988^[90]. However, while prototype systems are in development at number of research centers, commercial products, detailed examples and extensive documentation are unavailable in the open literature.

II.D.2 Knowledge-Based Approaches

In the same paper which laid the groundwork for the geometric abstractions presented above, Allwright^[90] investigated the utility of linking grid point positioning and spacing throughout the mesh to generic features of the configurations. Such concepts lead to so-called knowledge-based approaches and similar procedures have been investigated by a number of researchers and commercial firms. Applications of knowledge-based approaches revolve around the idea that a new configuration should be blocked only once and subsequent meshes for topologically similar configurations may then make use of similar block layouts and point distributions.

While conceptually simple, the ability to grid a new configuration from a library of previously studied examples is of indispensable utility in a detail design or loads estimation environment. For example, adjusting the position or contour of a pylon/nacelle combination, or investigating various flap deflections are common operations which should not require the re-blocking of entire mesh systems. The current software releases of many mesh generation software systems include this functionality. Often described as “macros”, “configuration libraries” or “checkpointing” such approaches all permit the engineer to rapidly grid topologically similar cases with a minimum of new input.

The concept of “driving” mesh generation systems from an experience base has been extended by Buning^[25] who has investigated prototype software which generates entire mesh systems, automatically runs flow simulations, and then extracts pre-determined results from the discrete solutions. These software “scripts” run at the operating system level and call FORTRAN or C codes which search for certain geometric or topological features in the input geometry. The system has been demonstrated for wing-body geometries using the OVERFLOW^[91] solver and the Hypgen^[92] mesh generation systems and is being

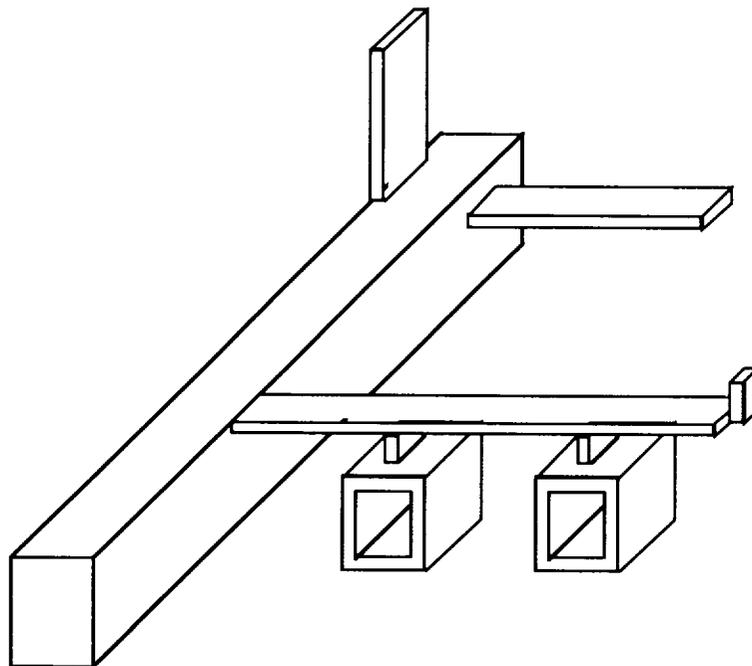


Figure 10. Sample blocked abstraction of HWT configuration following references [90] and [85].

considered for extension to more general classes of problems.

Extensions of these concepts have motivated research into controlling the blocking and point distribution process with expert systems or artificial intelligence. Preliminary work in this direction was performed by Dannenhoffer in 1991^[93] and the topic was re-visited in a NASA sponsored workshop in late 1994^[94]. Within such a framework, expert systems would either choose from libraries of pre-defined blocking strategies, or create new block layouts according to a pre-defined rule base.

II.D.3 Algorithmic Approaches

In the introduction of a paper which proposes a possible theoretical framework for formalizing the problem of multiple block structured grid generation, Cordova^[89] poses the following question:

“Could it be that human intervention is necessary for solving the blocking problem?”

When one considers that unstructured mesh generators routinely operate with only minimal user input, the obvious answer to this question must be that such intervention can not be necessary. This argument is illuminating because it highlights the difference between our approach to multi-block gridding and techniques for unstructured mesh generation. Just as mesh blocking is a domain decomposition into hexahedra, unstructured mesh generation is a decomposition into tetrahedra.

Following this reasoning, Cordova suggests viewing structured grid blocking as a problem in Topological Graph Theory^[95]. Within this framework, one seeks a hexahedral tessellation of the computational domain, in which each cell constitutes a grid block - or even a further decomposition to yield individual grid cells. Using the language of Computational Geometry (CG), the unstructured and structured mesh generation problems may be phrased as follows:^[89]

Unstructured Mesh Generation - Decompose a region bounded by algebraic surfaces of degree = 1 into tetrahedra.

Block Structured Mesh Generation - Decompose a region bounded by algebraic surfaces of degree ≥ 1 in to deformed hexahedra.

The inequality in the second statement recognizes that a block on a wall boundary will not necessarily have planar faces. We note in passing that unlike tetrahedra, hexahedra are not rigid figures (polyhedra). This property, and the inequality in the second statement leads to the observation that while the CG problem of tetrahedral meshing is linear, the block structuring problem is non-linear, and solution strategies may require linearization steps.

With these definitions, Ref. [89] suggests the following algorithm for approaching multiple block grid generation:

- Algorithm II:**
- a. Approximate the configuration with polyhedra.
 - b. Decompose surrounding space into convex polyhedra.
 - c. Construct computational space.
 - d. Set boundary conditions in each block
 - e. Solve elliptic equations globally over computational space.

Before examining this algorithm, we return to a brief analysis of the complexity of operations involved in the construction of tetrahedral grids as discussed in section II.A. The process of triangulating given coordinate data has linear complexity - that is to say that the operation count required to triangulate N vertices scales linearly with the number of vertices. Sorting N vertices requires $O(N \log N)$ operations and procedures such as clipping or the removal of hidden surfaces have quadratic complexity ($O(N^2)$). Such processes are in the class P since they may be performed in *polynomial time*. Other processes cannot be performed in polynomial time, and in fact their complexity is undetermined. These problems are said to belong to the class NP if candidate solutions may be verified in polynomial time^{[89][96]}. A problem is

termed NP-complete when one can show that if it *could* be solved in polynomial time then all similar NP problems could also be solved with similar complexity.

In applying the decomposition step (b) of Algorithm II to 2-D blocking problems, one finds that decomposition of a multiply connected region (a polygon with polygonal holes in it) into quadrilateral regions is NP-complete^[97]. It is interesting to note that if the region happens to be simply connected (no holes) it may be quadrilateralized in polynomial time. However, NP-completeness does not imply that the problem is unsolvable, simply that one must search for a novel algorithm for generating candidate solutions. For example, a novel algorithm for quadrilateral decompositions in the plane may be obtained by first obtaining a candidate solution through a Delaunay triangulation of the coordinate data (possible in linear time). Then one simply inserts new sites at the centroids of each triangle, and connects them to new sites inserted at the midpoint of each edge. The result will always be a quadrilateral decomposition of the region, despite the problem's inherent difficulty due to its NP-completeness^[98]. Unfortunately, decompositions constructed through this method are unlikely to meet the requirements of mesh smoothness that are imposed by the relatively low-order (linear) elements used by most CFD solvers.

In reference [99] Cordova suggests pursuing similar novel algorithms for solving the blocking problem. In addition the work in references [89] and [100] show that the framework and strategies of computational geometry and the theory of NP-completeness may be applied to the problem of point placement which is necessary to construct the computational space (step c, algorithm II) for a given mesh.

These examples demonstrate that while research in algorithmic approaches to solving the blocking and point placement problems is still formative, the work is promising and worthy of pursuit. Significant theoretical resources have been developed in the fields of Topological Graph Theory and Computational Geometry which are directly applicable to the problems of constructing structured multi-block meshes. Such approaches offer the possibility of replacing the heuristics of interactive mesh generation with robust, provable techniques.

III. OPPORTUNITIES IN DESIGN AND APPLIED AERODYNAMICS

With the partial review of emerging CFD tools in the preceding section, focus now shifts to the application of these techniques within design and applied aerodynamics environments. Throughout this discussion a recurring theme will be the appropriateness of physical modeling and numerical tools as measured by the temporal requirements of the process. The examples and arguments in the introduction (sect. I), point out that various aspects of design and applied aerodynamics require a different balance of speed and accuracy. In reference [21] Rubbert uses an economic model of the aircraft industry to make a strong case that often a faster, but more approximate, process may lead to a higher level of functional "goodness" if that process is time constrained. In economic terms, he notes that maximum market share is generally achieved before a process has had time to reach its asymptotic level of functional goodness. Ultimately, the time asymptotic level of goodness achievable by a given process may mean very little if it takes too long to get close to that level. In the context of numerical simulations, the speed of a process is related to set-up time, CPU/memory requirements and post processing of discrete solutions.

As various CFD technologies mature, set-up and execution times will content to decrease. Thus, increasingly sophisticated physical modeling will become available progressively earlier in the design cycle. The discussion in this section is intended, therefore, to reflect the status of CFD research and computing hardware that currently exists.

III.A Inviscid Techniques

Since they require minimal set-up, and avoid the problems of laying out and constructing a block structured mesh, unstructured and Cartesian approaches currently offer an attractive method for computing inviscid simulations. For example the unstructured, LWT simulation that was presented in figure 4 demonstrated that

a one million node Euler simulations currently require between one and two hours of supercomputer CPU. Alternatively, 200,000 vertex simulations currently converge in less than an hour on modern engineering workstations. The documented success of these methods suggests that 3D unstructured or Cartesian Euler solvers could be coupled with strip boundary layer modeling to provide an extremely flexible platform for rapid aerodynamic analysis. Such a system may be of greater immediate use than a full unstructured Navier-Stokes scheme, since inviscid solutions require approximately an order of magnitude fewer nodes, and 5-10 times less storage and CPU. Moreover, the discussion in section II points out that there are many unresolved issues remaining in the extension of these techniques to viscous simulations, while the inviscid technology is relatively mature.

Readily accessible inviscid, or inviscid and boundary-layer analysis is extremely valuable in various aspects of design and applications aerodynamics. The task of preliminary loads estimation is followed in series by that of structural analysis. This serial connection, and the importance of the follow-on task makes load estimation an example of a time critical process which would benefit greatly from improved physical modeling. Although it is true that many critical loading conditions occur in regions of the flight envelope in which Euler and boundary layer modeling may not be entirely sufficient, the approach offers great improvements over current approaches which are largely based on panel and potential codes. With the ability to capture non-linear features in the flow, Euler modeling permits prediction of vortex trajectories and shock structures which impact component loads over a wide range of conditions. With finer mesh resolution or solution adaptation, the same system would also provide a valuable method for getting preliminary results for detailed design or propulsion integration (PI) studies. With much of the set-up already completed during loads estimation, these fine mesh runs would guide PI teams in developing high quality block structured grid systems for full Navier-Stokes simulations. This preliminary analysis would alert design engineers to possible unfavorable shock or vortex interactions and permit *a priori* tailoring of the viscous mesh.

A critical aspect of such an approach is integration. Many manufacturers already use some kind of Euler-boundary layer modeling, but typically such systems are constrained by excessive overhead and are frequently poorly integrated. Automated mesh generation, macros for running angle of attack sweeps, automated post-processing, etc. are all features central to the usefulness of such a system. The engineer must be given quick access to the important data and the ability to assess the quality of the discrete solutions.

III.B Surface Modeling and Preliminary Design

In surveying the techniques discussed in Section II, deficiencies in surface modeling emerged as an impediment to nearly all techniques for applied CFD. In this context, "surface modeling" refers to the process of generating an accurate representation of the true geometry in a form directly amenable to mesh generation. Unstructured, Cartesian and prismatic approaches require constrained triangulations of either the aircraft's surface or individual components. Block structured solvers require *i,j* ordered surface meshes. Robust techniques for generating these descriptions have been documented in the literature, and are generally coupled with mesh generation software. However, transitioning this task to preliminary design or CAD software may offer significant advantages. Multi-disciplinary preliminary design and CAD systems contain a complete description of the geometry, and provide access to this information for all subsequent disciplines in the analysis cycle. Importing a CAD file into a mesh generation system places the burden of surface modeling not only on the mesh generator, but also on the CAD engineer. Since it is unlikely that the CFD model will include the myriad of details that the full CAD model includes, the model will need to be "cleaned-up" within the CAD system. Moreover, if a configuration originates in a preliminary design system, this model must first be translated to a dedicated CAD system before production of a model for grid generation. The intermediate step is unnecessary. With surface triangulations or surface grids directly available as output options from preliminary design or CAD software the current, updated geometry would be immediately available for aerodynamic analysis.

Preliminary design is an inherently multi-disciplinary process. A variety of software is in use, and many include multi-disciplinary optimization strategies that “fly” candidate vehicle designs through specified mission profiles, permitting multi-point optimization and trade-off analysis^{[101],[102],[103]}. Physical modeling in these systems is based on an internal representation of the geometry, and includes approximate methods for structures, manufacturing and aerodynamics. The requirement for speed drives such systems to use handbook, or panel methods for approximate aerodynamic analysis. If such systems were extended to output constrained surface triangulations it would then be possible to also spawn coarse grid, Euler or full-potential CFD solutions using either unstructured or Cartesian meshes. Critical points within the flight envelope could be identified and run within the preliminary design environment, offering improved physical modeling. Preliminary design could then operate further from its experience base to explore novel configurations while simultaneously increasing the confidence in the results obtained with lower order methods.

III.C Navier-Stokes Analysis

Processes such as wing optimization, propulsion integration, and high-lift system design ultimately require full Navier-Stokes analysis. Although the technology is constantly changing, block structured techniques currently appear to offer the most confidence and efficiency for such analysis. Many recent research programs have been dedicated to improving the efficiency of the grid generation process. The review of current and prospective block structuring techniques in Section II permits general statements concerning structured grid generation. Specifically, while developers strive for general algorithmic solutions to domain decomposition and point placement problems, interactive approaches have yielded remarkable success for specific topologies. Various research efforts have produced streamlined interactive procedures which guide even novice users through the block generation process using minimalist descriptions of complex aerospace geometries. Such approaches have demonstrated mesh set-up times on the order of hours using engineering workstations. Meanwhile, experience with these and more traditional mesh generation systems has resulted in a large knowledge base, or library, linking various mesh block arrangements to specific surface topologies. Work has also explored using this knowledge base to mesh new configurations by deforming topologically similar blockings from previous, or generic configurations. Clearly the possibility exists to combine these factors into grid generation systems which are vastly improved over those currently available.

If surface meshes were available directly from CAD products, or from prior steps in design, such mesh generation systems would offer greatly reduced set-up times. Even if appropriate surface meshes are not available, the use of abstractions discussed in Section II would permit block layout to be performed in parallel with surface mesh preparation - reducing at least the calendar time required for multi-block structured analysis.

Off-design analysis, wing optimization, and various tasks in applied CFD require the consideration of a variety of specific cases. This fact underscores the necessity of integrated CFD software to accept macroscopic automation. Automated control of job submittal, job monitoring and routine post-processing would again lead to more efficient processes.

IV. CONCLUSIONS

This paper has reviewed the progress of a variety of emerging technologies against the issues which limit CFD’s usefulness in design and applied aerodynamics. In doing so, the discussion has concentrated on approaches which intend to reduce the overhead associated with flow simulations around complex configurations. The discussion examined the current status of unstructured, hybrid, and Cartesian approaches as well as techniques for automating traditional multi-block mesh generation schemes. These methods were evaluated with special consideration of the differences that exist between research and design environments.

The rapidly maturing state of unstructured and Cartesian based techniques suggests the possibility of highly automated Euler-boundary layer solvers for use in loads estimation and other time-critical processes within the design cycle. Mesh generation for these techniques is largely automated and it requires only the generation of constrained triangulations for surface modeling as an input. Similarly, opportunities exist to increase the level of automation in the construction of blocked meshes for use with structured Navier-Stokes solvers. Strategies which link block structured mesh generation algorithms with libraries of prior examples are particularly attractive, since they avoid repeated meshing of topologically similar configurations. The interactive approach of building grid abstractions is also promising since it permits the blocking process to go on in parallel with surface modeling efforts. Work in algorithmic based auto-blocking suggests that domain decomposition and point placement operations in multi-block mesh generation may be properly posed as problems in Computational Geometry. This approach is unifying since it describes both multi-block structured methods and unstructured mesh methods within a common framework.

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